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# Rapid Acquisition for Direct Sequence Spread-Spectrum Communications Using Parallel SAW Convolvers

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*Abstract*—In this paper, a technique is described which uses multiple surface acoustic wave (SAW) devices in parallel to reduce the acquisition time of a direct sequence spread-spectrum communication system. Analysis of system performance in both the search and lock modes is presented, and key quantities such as probability of false alarm, probability of correct detection, mean dwell time, and mean time to lose lock are derived.

## I. Introduction

In this paper, a rapid acquisition technique, based upon the use of parallel processing the received waveform of a direct sequence spread-spectrum communication system with multiple surface acoustic wave (SAW) convolvers, is presented. Because the processing is indeed done in parallel, a decrease in acquisition time from a comparable serial search scheme is achieved.

A derivation of the probabilities of false alarm and correct detection for both the search mode and the lock mode are presented, as well as a Markov chain analysis of an overall search-lock strategy. Expressions are derived for such key system parameters as probability of entering lock and average time to loss of lock. Finally, some numerical results are presented, showing the variation of false alarm and detection probabilities with the number of parallel convolvers and with the ratio of received energy per bit to noise spectral density.

The paper is divided into six sections, with a description of the system presented in the next section, the derivations of probability of false alarm and probability of correct detection presented in Section III, a Markov chain analysis of the search and lock modes' behavior given in Section IV, and numerical results given in Section V. Conclusions are then presented in the last section.

## II. System Description

The acquisition scheme can be best described by referring to Fig. 1. There are two modes of operation, a search mode and a lock mode. Consider the search mode first, during which all switches are in position 1. (Ignore, for the moment, the recirculating delay line circuit at the input to the system.) Notice that there are $N$ convolvers, each of duration $2T$ seconds, where it is assumed that $M$ chips of the spreading sequence span $T$ seconds. The full period of the spreading sequence, $L$, is divided into subsequences each of length $M$, and for simplicity, it is assumed that $L/M$ is an integer.

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Each of the $N$ convolvers has as a reference input one of the subsequences of length $M$. Initially, let us assume that the total phase uncertainty at the start of acquisition, say $K$ chips, is spanned by the $MN$ phase positions of the $N$ convolvers. (Notice that even if $L$ is much greater than $MN$, this uncertainty condition could still be satisfied.) This guarantees that the correct phase position of the received waveform is "seen" by one of the $N$ convolvers somewhere in the integration interval of the convolvers.

The convolver outputs are sampled at times $t = T + jT_c$, where $T = MT_c$ and $j = 0, 1, \cdots, M-1$. The length of each convolver is chosen to be $2T$. With these parameter values, if the $N$ reference inputs enter the convolvers at $t = 0$, the unknown phase position will overlap the correct phase reference of one of the $N$ convolvers at some time during the interval $t \in [MT_c, (2M-1)T_c] = [T, 2T - T_c]$. After sampling, the largest of the resulting $MN$ samples is chosen as the correct phase of the incoming waveform. Since one of the $MN$ samples is the correct sample, it is therefore *possible* to initially acquire in $2T$ seconds. The actual mechanism for making the final decision regarding acquisition will be described in the next section.

Suppose now that the initial phase uncertainty $K$ is greater than $MN$. For this case, the recirculating delay line, shown in Fig. 1, is used at the input to the system with both switches $A$ and $B$ closed. On the input side, after the first $T$-second segment of the received waveform is in the system, switch $A$ is opened and the $T$-second segment is recirculated $J$ times, where $J$ equals $\lceil K/MN \rceil$, and $\lceil x \rceil$ is the smallest integer greater than or equal to $x$. On the reference side, the first $N$ subcodes are put through the respective convolvers during the interval $[0, T]$. From $[T, 2T]$, no reference input is used, but from $[2T, 3T]$, the next $N$ subcodes are used as reference inputs. In general, the $i$th set of $N$ subcodes, $i = 1, \cdots, J$, is used as reference inputs during the interval $[(i-2)T, (i-1)T]$. For simplicity, in most of what follows, $K$ is taken equal to $MN$, and hence $J = 1$.

Considering next the lock mode, switch $A$ is kept closed, switch $B$ is kept open, and all other switches that were in position 1 are now put in position 2, so that each convolver output passes through the appropriate delay line. (Note that the recirculating delay line at the convolver input is not used.) In this mode of operation, the $N$ outputs are noncoherently summed to yield an effective increase in integration time over that used in the search mode. To reset the reference input during the lock mode simply requires separating successive subcodes into any of the $N$ convolvers by $NT$ seconds.

Notice that when the system is locked to the correct phase position, all the inputs to the delay lines have been despread, and hence, the delay lines themselves do not have to be broadband devices. Similarly, if an incorrect phase position is being observed, while the inputs to the delay lines will no longer be despread, using narrow-band delay lines will still not result in any performance degradation, since under this latter condition, a signal component out of the delay lines is not wanted.

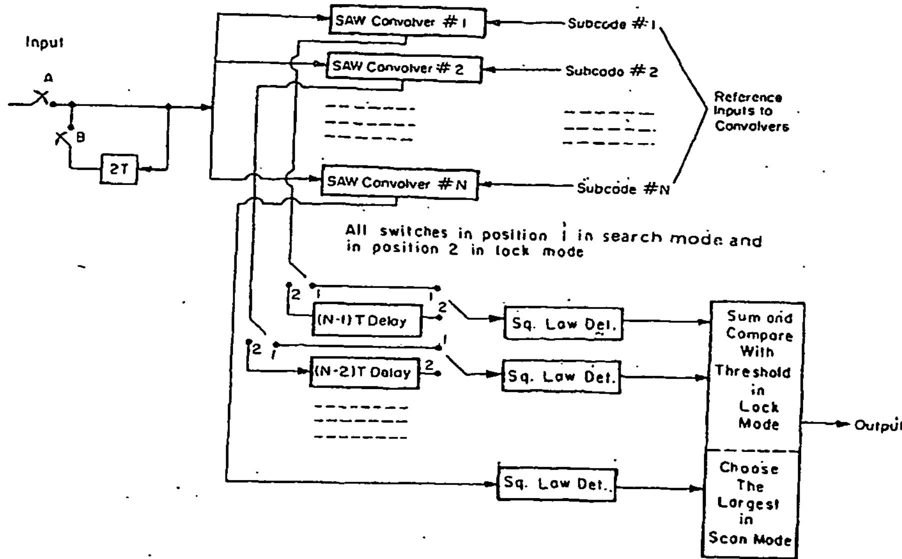


Fig. 1. SAW-implemented synchronization scheme for spread-spectrum receiver.

## III. Performance Analysis

Let the reference input to the $i$th convolver be denoted by $PN_{(i-1)M+1}(t)$, where $PN_k(t)$ denotes a segment of a longer pseudonoise sequence [denoted PN $(t)$] from $(k-1)T_c$ to $(k-1+M)T_c$, $M$ is the number of chips in each subcode, and $T_c$ is the duration of each chip. Therefore, $PN_{(i-1)M+1}(t)$ is a subcode in the interval $[(i-1)MT_c, iMT_c]$, $i = 1, 2, \cdots, N$. For simplicity in notation in what follows, the reference subcode $PN_{(i-1)M+1}(t)$ will be denoted by $PN_i^{(R)}(t)$. (Note that we have artibrarily defined the start of subcode $PN_1^{(R)}(t)$ to correspond to $t = 0$.)

To determine the probability that the correct phase position has been found, assume that $PN_j(t)$ is the subcode that is currently being received, and to be specific, assume the received subcode $PN_j(t)$ plus noise enters the left side of the convolvers of Fig. 1 at the same instant of time that the reference subcodes $PN_i^{(R)}(T-t)$, $i = 1, \cdots, N$, enter the right side. (Note that by making the convolver length equal to $2T$, it is not *necessary* to have synchronism between the start of a received subcode and the start of the reference codes; it is done here only for convenience.) Also, for simplicity, assume $J = 1$. If we concentrate on the $i$th convolver, then from [1], it is straightforward to show that at any time $t \in [T, 2T]$, the convolver output is given by

$$U_i(t) = \int [A\,PN_j(\tau)\cos w_0\tau + n(\tau)] \cdot PN_i^{(R)}(-2t + T_1 + T + \tau) \cdot \cos[w_0(\tau - 2t) + \phi]\, d\tau \tag{1}$$

where $T_1$ is the length of the convolver, $A$ is a constant amplitude, $\phi$ is a random phase uniformly distributed in $[0, 2\pi]$, and $n(t)$ is AWGN of two-sided spectral density $\eta_0/2$. The specific limits of integration of (1) depend upon the degree of overlap between the received waveform and the reference code. If we assume the received waveform has completely occupied the convolver when the subcodes first enter, then for $t \in [T, 2T]$, the limits are 0 to $T$.

If double frequency terms are ignored in the integrals and if $T_1$ is set equal to $2T$, then (1) reduces to

$$\begin{aligned} U_i(t) = \frac{1}{2}\int_0^T & A\,PN_i^{(R)}(-2t + 3T + \tau)\,PN_j(\tau)\,d\tau \\ & \cdot \cos(2w_0t - \phi) + \int_0^T PN_i^{(R)}(-2t + 3T + \tau) \\ & \cdot \cos(w_0\tau)n(\tau)\,d\tau \cos(2w_0t - \phi) \\ & + \int_0^T PN_i^{(R)}(-2t + 3T + \tau) \\ & \cdot \sin(w_0\tau)n(\tau)\,d\tau \sin(2w_0t - \phi). \end{aligned} \tag{2}$$

Assuming initially that $i = j$ and that the sampling time $T_s$ corresponds to precisely sampling the correct convolver at the peak of its response, then

$$U_j(T_s) = [\tfrac{1}{2}AT + N_{c_j}(T_s)]\cos[2w_0T_s - \phi] + N_{s_j}(T_s)\sin[2w_0T_s - \phi] \tag{3}$$

where

$$N_{c_j}(T_s) \triangleq \int_0^T PN_j^{(R)}(\tau)n(\tau)\cos w_0\tau\, d\tau \tag{4}$$

and

$$N_{s_j}(T_s) \triangleq \int_0^T PN_j^{(R)}(\tau)n(\tau)\sin w_0\tau\, d\tau. \tag{5}$$

Both $N_{c_j}(T_s)$ and $N_{s_j}(T_s)$ are zero-mean Gaussian random variables with variance $\eta_0 T/4$. They are also statistically independent.

For $i \neq j$, the output of the convolver at the sampling instant is given by

$$U_i(T_s) = [\tfrac{1}{2}AR_{ij} + N_{c_i}(T_s)]\cos[2w_0T_s - \phi] + N_{s_i}(T_s)\sin[2w_0T_s - \phi] \tag{6}$$

where $N_{c_i}(T_s)$ and $N_{s_i}(T_s)$ have the same statistical properties as $N_{c_j}(T_s)$ and $N_{s_j}(T_s)$, and where

$$R_{ij} \triangleq \int_0^T \mathrm{PN}_i^{(R)}(t)\,\mathrm{PN}_j(t)\,dt. \tag{7}$$

Finally, with an appropriate change of notation, (6) can be seen to apply to the output of any of the $N$ convolvers at any of the $M-1$ *incorrect* sampling times. Therefore, if the output of the $i$th square-law detector at the $k$th sampling time is denoted by $S_i(T+kT_c)$, and if the correct sampling time (which equals $3T/2$ for the specific timing being used here) corresponds to $k=k_c$, then an error will occur if

$$S_i(T+kT_c) > S_j(T+k_cT_c) \qquad \text{for any } k,\ i \neq j,$$

or if

$$S_j(T+kT_c) > S_j(T+k_cT_c) \qquad \text{for } k \neq k_c.$$

These conditions correspond to an incorrectly matched convolver at any sampling time having an output larger than that of the correct convolver at the correct sampling time, and to the correct convolver having a larger output at some time other than the correct sampling time, respectively.

Denoting by $\{S_m\}$, $m = 1, \cdots, MN-1$, the set of samples of either $S_i(T+kT_c)$ for any $k$ or $S_j(T+kT_c)$ for $k \neq k_c$, and denoting $S_j(T+k_cT_c)$ by $S_0$, then an error occurs if $S_m > S_0$ for any $m$. Hence, the probability of error in the search mode, $P_e^{(s)}$, can be upper bounded by

$$\begin{aligned} P_e^{(s)} &= P\{(S_1 > S_0) \text{ or } (S_2 > S_0) \text{ or } \cdots \text{ or } (S_{MN-1} > S_0)\} \\ &\leqslant \sum_{i=1}^{MN-1} P\{S_i > S_0\}. \end{aligned} \tag{8}$$

If $P_{ij} \triangleq P\{S_i > S_j\}$, then $P_{ij}$ can be shown to be given by [2]

$$P_{ij} = \tfrac{1}{2}\left[1 - Q(\sqrt{b}, \sqrt{a}) + Q(\sqrt{a}, \sqrt{b})\right] \tag{9}$$

where

$$\begin{aligned} \begin{Bmatrix} a \\ b \end{Bmatrix} &\triangleq \frac{1}{2}\left[\frac{2\sigma^2(A^2R_{ij}^2 + A^2T^2) - 4\sigma^2\rho_{ij}A^2R_{ij}T}{(2\sigma^2)^2 - 4(\rho_{ij}\sigma^2)^2} \right. \\ &\qquad \left. \pm \frac{A^2R_{ij}^2 - A^2T^2}{[4\sigma^4(1-\rho_{ij}^2)]^{1/2}}\right] \\ &= \left(\frac{AT}{2\sigma}\right)^2 [1 \mp (1-\rho_{ij}^2)^{1/2}] \end{aligned} \tag{10}$$

$$Q(a,b) \triangleq \int_b^\infty x e^{-(a^2+x^2)/2} I_0(ax)\,dx \tag{11}$$

$$\sigma^2 \triangleq \eta_0 T \tag{12}$$

$$\rho_{ij}\sigma^2 \triangleq \eta_0 R_{ij} \tag{13}$$

and $I_0(x)$ is the modified Bessel function of the first kind and zeroth order.

As a special case, if all the $\rho_{ij}$ are approximately zero, then $P_{ij}$ reduces to

$$P_{ij} = \tfrac{1}{2} e^{-E/2\eta_0} \tag{14}$$

and (8) reduces to

$$P_e^{(s)} \leqslant (NM-1)\tfrac{1}{2} e^{-E/2\eta_0}. \tag{15}$$

In (14) and (15), $E = A^2T/2$ and is the energy per integration interval, $T$, of the transmitted signal. As a final observation for the search mode analysis, note that for this special case (all $\rho_{ij} = 0$), the exact probability of error can be found [3] and is given by

$$\begin{aligned} P_e^{(s)} = 1 - \sum_{i=0}^{NM-1} \binom{MN-1}{i} \frac{(-1)^{NM-1-i}}{NM-1} \\ \cdot \exp\left[-\frac{E}{\eta_0}\,\frac{MN-i-1}{NM-i}\right]. \end{aligned} \tag{16}$$

Consider now the lock mode, in which all the switches in Fig. 1 are set at position 2. In this latter mode, the $N$ convolver outputs are all appropriately delayed and summed, but only at the specific phase position decided upon in the search mode.

To analyze the performance of the system in the lock mode, the approximation that all the $\rho_{ij} = 0$ is made in order to simplify the analysis. With this approximation, it can be seen that when an incorrect phase position is being observed, the decision variable has a central chi$^2$ distribution with $2N$ degrees of freedom, and when the correct phase position is being observed, the decision variable has a noncentral chi$^2$ distribution with $2N$ degrees of freedom and noncentrality parameter $a_1^2 = NA^2T^2$. Therefore, the probabilities of false alarm and correct detection in the lock mode are given by

$$P_{fa}^{(l)} = \int_\gamma^\infty \frac{1}{(2\eta_0 T)^N \Gamma(N)}\, y^{N-1} e^{-y/2\eta_0 T}\,dy \tag{17}$$

and

$$\begin{aligned} P_d^{(l)} = \int_\gamma^\infty \frac{1}{2\eta_0 T}\left(\frac{y}{a_1^2}\right)^{(N-1)/2} e^{-(y+a_1^2)/2\eta_0 T} \\ \cdot I_{N-1}\left(\frac{a_1\sqrt{y}}{\eta_0 T}\right) dy \end{aligned} \tag{18}$$

respectively, where $\gamma$ is the threshold, $\Gamma(\cdot)$ is the gamma function, and $I_{N-1}(\cdot)$ is a modified Bessel function.

## IV. Search Mode and Lock Mode Transitions

In order to complete both the system description and the performance analysis, the specific procedure for transitioning from one phase position to the next in the search mode must be specified, as well as the procedure for transitioning from the search mode to the lock mode. Towards this end, a search-lock strategy similar to that described in [4] will be used. In particular, once a phase position has been decided upon in the search mode, two more successive "hits" at that phase position will be required in order to enter into the lock mode. That is, once all phase positions within the uncertainty range have been examined and an initial decision made as to which is the correct position, the procedure is repeated two more times. If the same relative phase position is chosen both times, the system advances to the lock mode. Similarly, two successive "misses" in the lock mode will be necessary in order for that phase position to be rejected and the search mode reinitiated. However, in the lock mode, only the specific phase position previously decided upon in the search mode is examined, not all possible phase positions falling within the uncertainty region.

A flow diagram of this procedure is shown in Fig. 2. It is well known (see, e.g., [4], [5]) that this type of process can

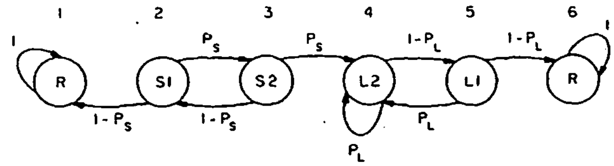


Fig. 2. Markov chain model of search/lock strategy.

be modeled as a finite Markov chain with absorbing boundaries. From Fig. 2, it is seen that there are six states to this particular Markov chain, labeled 1–6. States 1 and 6 are designated $R$ for "rejection," meaning when either state is reached, the phase position currently being examined is rejected (i.e., the Markov chain has reached one of its absorbing states). States $S_1$ and $S_2$ correspond to the two search mode states, whereas $L1$ and $L2$ correspond to the two lock mode states.

In Fig. 2, $P_s$ denotes either $P_e^{(s)}$ or $1 - P_e^{(s)}$ (depending upon whether the phase under consideration is incorrect or correct, respectively) and $P_L$ represents $P_{fa}^{(l)}$ or $P_d^{(l)}$ under similar circumstances for the lock mode. Using the absorbing Markov chain model for this system, the canonical form for the transition matrix, described in [4] and [5], is shown below.

$$\bar{\bar{P}} = \begin{array}{c} \\ 1 \\ 6 \\ \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cc:cccc} 1 & 6 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{cc:cccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \hdashline 1-P_s & 0 & 0 & P_s & 0 & 0 \\ 0 & 0 & 1-P_s & 0 & P_s & 0 \\ 0 & 0 & 0 & 0 & P_L & 1-P_L \\ 0 & 1-P_L & 0 & 0 & P_L & 0 \end{array}\right] \end{array}$$

$$\triangleq \left[\begin{array}{c:c} \bar{\bar{S}} & \bar{\bar{\phi}} \\ \hdashline \bar{\bar{R}} & \bar{\bar{Q}} \end{array}\right]. \tag{19}$$

Forming the matrix

$$\bar{\bar{I}} - \bar{\bar{Q}} = \begin{bmatrix} 1 & -P_s & 0 & 0 \\ (1-P_s) & 1 & -P_s & 0 \\ 0 & 0 & 1-P_L & -(1-P_L) \\ 0 & 0 & -P_L & 1 \end{bmatrix} \tag{20}$$

where $\bar{\bar{I}}$ is an identity matrix, and defining the new matrix

$$\bar{\bar{B}} = [\bar{\bar{I}} - \bar{\bar{Q}}]^{-1}\bar{\bar{R}},$$

it is shown in [4] that the probability of entering lock is given by entry $b_{26}$ of the above $\bar{\bar{B}}$ matrix. Notice that the notation $b_{26}$ refers to elements of the $\bar{\bar{R}}$ matrix. That is, the rows of $\bar{\bar{B}}$ are numbered 2, 3, 4, and 5, and the columns of $\bar{\bar{B}}$ are numbered 1 and 6. For the system being considered here, the probability of entering lock is given by

$$b_{26} = \frac{P_s^2}{1 - P_s(1 - P_s)}. \tag{21}$$

In addition to the probability of entering lock, other quantities of interest are the mean dwell time in an incorrect phase position and the expected time to loss of lock, given that the system is in state $L2$ at the correct phase position. From [4] and [5], it is shown that if $\tau_j$ is the mean time to arrive at any absorbing state given an initial transient state $j$ (e.g., $S1$), then the vector of elements $\tau_j$, denoted $\bar{\tau}$, is given by

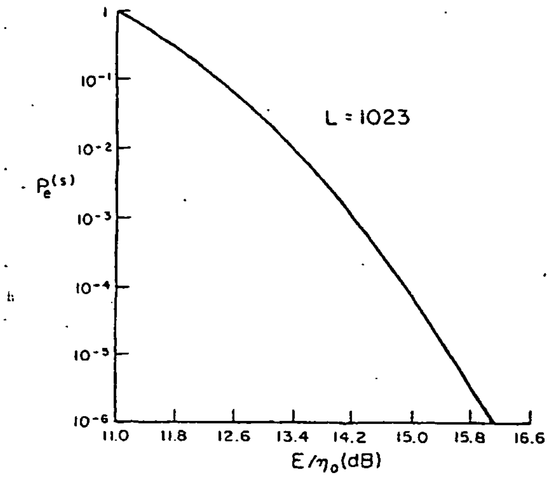


Fig. 3. Upper bound on probability of error in the search mode for $N = 11$ and $M = 93$.

$$\bar{\bar{\tau}} = \begin{bmatrix} \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_4 \end{bmatrix} = [\bar{\bar{I}} - \bar{\bar{Q}}]^{-1}\bar{\lambda} \tag{22}$$

where $\bar{\lambda}$ is a vector whose elements are the times needed to make each transition.

Since the time needed to move from one state to the next state for arbitrary $J$ is $2TJ$ in the search mode and $(N + 1)T$ in the lock mode, it can be shown in a straightforward manner that the mean dwell time is given by

$$\tau_2 = \frac{2J(1-P_L)^2(1+P_s) + (N+1)P_s^2(2-P_L)}{(1-P_L)^2[1-P_s(1-P_s)]}\, T \tag{23}$$

and the expected time to loss of lock is given by

$$\tau_4 = \frac{2-P_L}{(1-P_L)^2}\,(N+1)T. \tag{24}$$

## V. Numerical Results and Discussion

As a specific example, it will be assumed that a PN sequence of length $L = 1023$ is being received simultaneously in $N = 11$ convolvers, thereby resulting in $M = 93$ unknown phase positions in each convolver. If the approximation leading to (14)–(16) is used, namely, that all the $\rho_{ij}$ of (13) are zero, the curve of $P_e^{(s)}$ shown in Fig. 3 results. For the same system in the lock mode, Fig. 4 shows results of $P_d^{(l)}$ for various values of $\gamma$ (or equivalently for various values of $P_{fa}^{(l)}$).

The variations of $P_{fa}$ and $P_d$ with $N$ are illustrated in Figs. 5–9. Fig. 5 shows $P_{fa}^{(s)}$ versus $E/\eta_0$ for three values of $N$, $N = 5$, 10, and 15. Similar results for $P_d^{(l)}$ are presented in Fig. 6, and Figs. 7–9 show how $P_d^{(l)}$ varies with $P_{fa}^{(l)}$ for a given value of $N$.

As a perspective on the effect of approximating the $\rho_{ij}$ by zero for $i \neq j$, the exact $P_{ij}$ of (9) were computed, and some

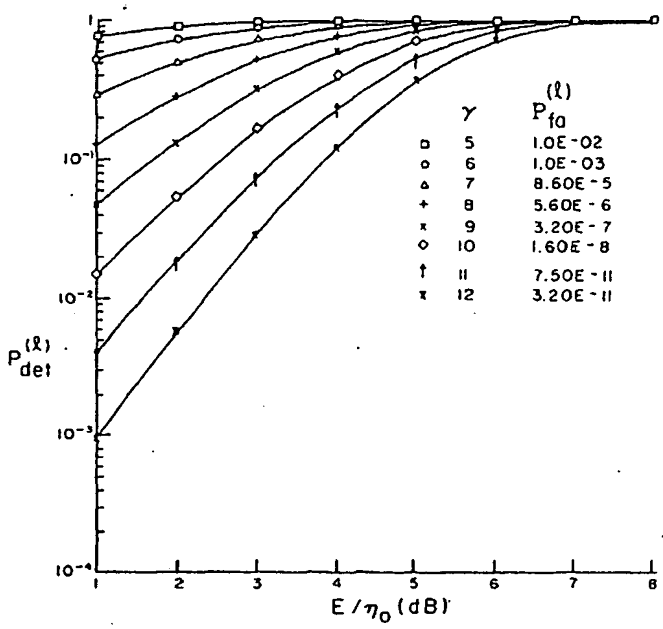


Fig. 4. Probability of detection versus $E/\eta_0$ for various values of threshold, for $N = 11$ and $M = 93$ in the lock mode.

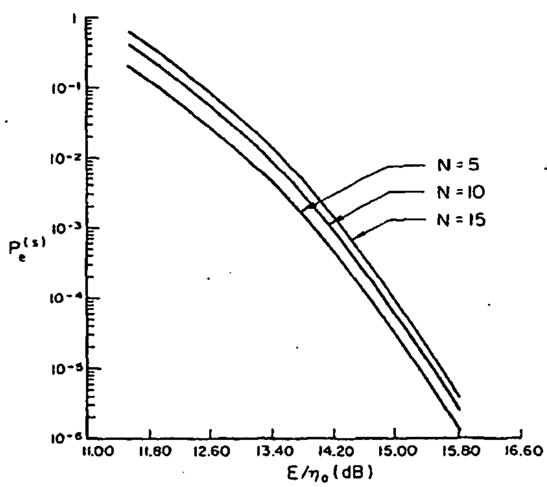


Fig. 5. Probability of error in the search mode for various $N$ and $M = 93$.

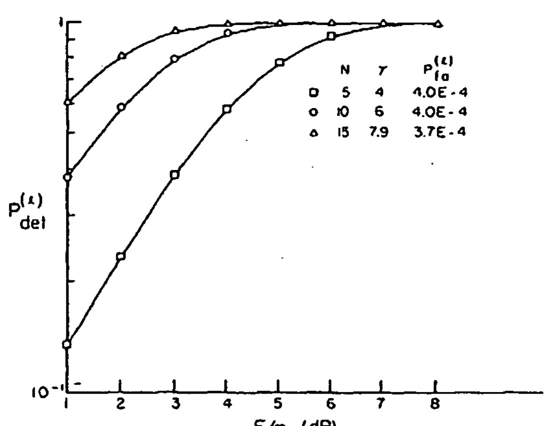


Fig. 6. Probability of detection in the lock mode for $N = 5, 10, 15$.

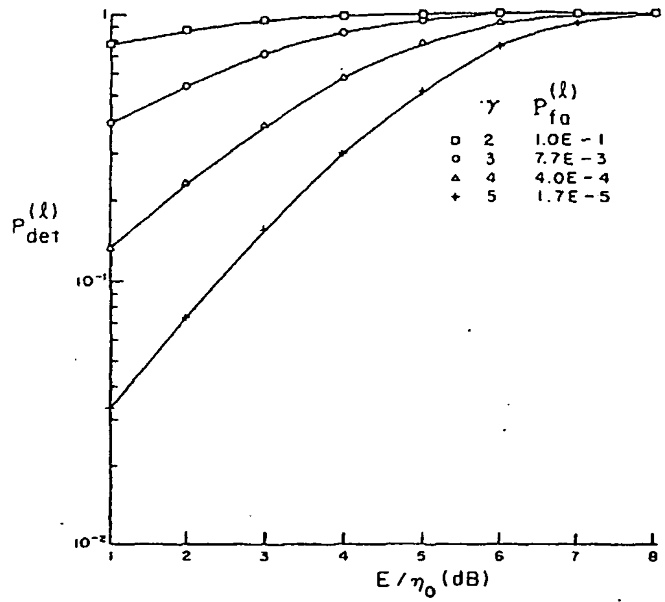


Fig. 7. Probability of detection for various thresholds in lock mode ($N = 5$).

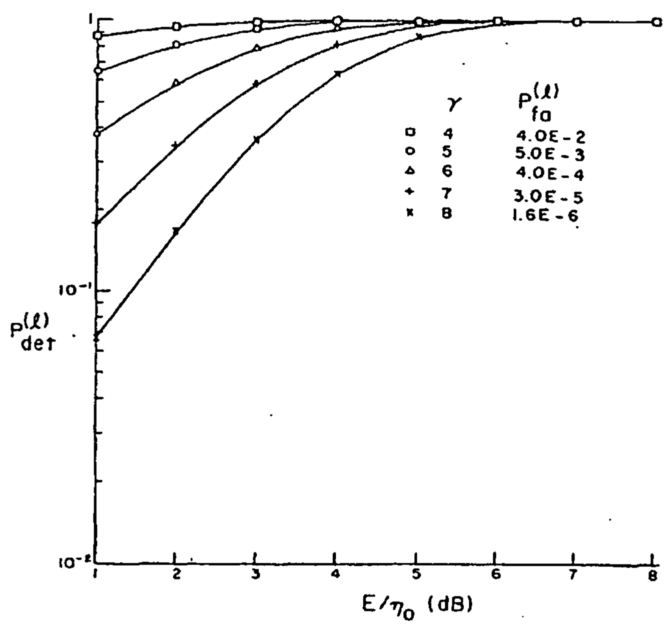


Fig. 8. Probability of detection for various thresholds in lock mode ($N = 10$).

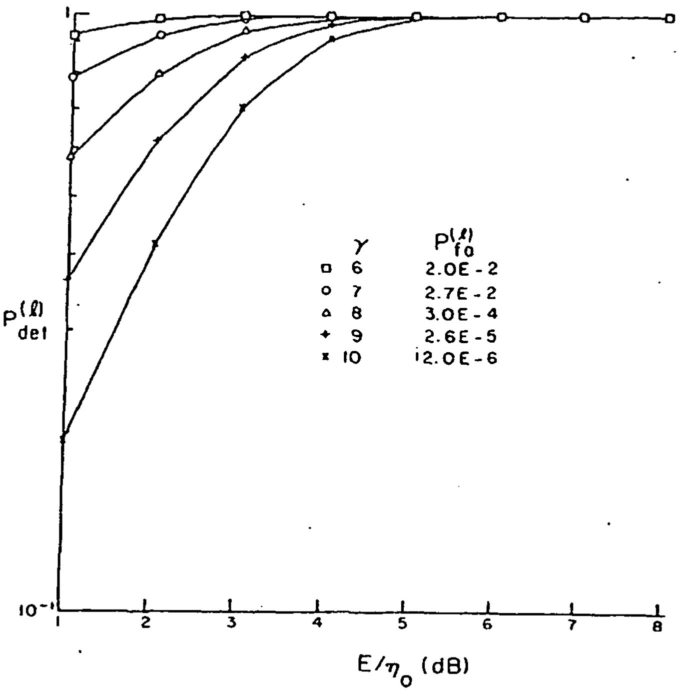


Fig. 9. Probability of detection for various thresholds in lock mode ($N$ = 15).

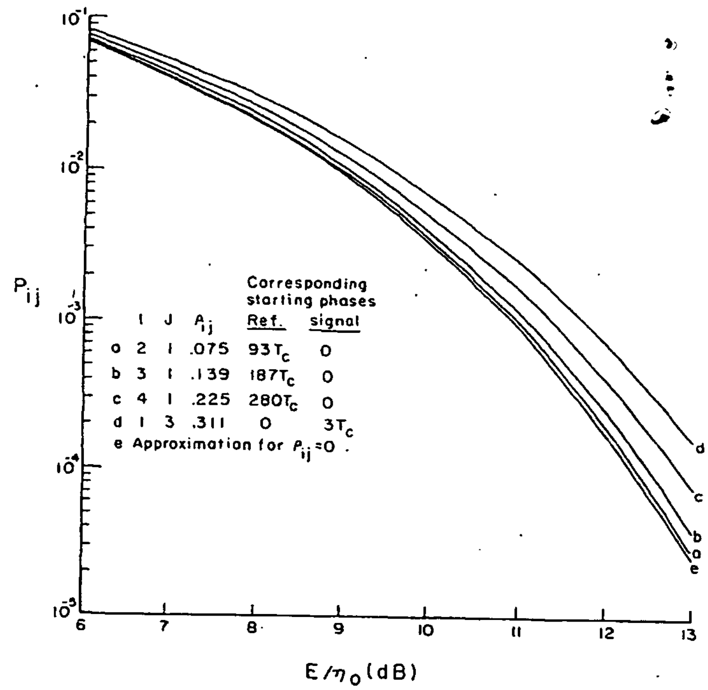


Fig. 10. Probability of error for various relative starting phases of reference and input for $N$ = 11 and $M$ = 93 in the search mode.

of the results are shown in Fig. 10. For the same PN sequence of length $L = 1023$ referred to above, a specific subsequence of length $M = 93$ was fed into all 11 convolvers and the resulting 1023 values of $\rho_{ij}$ were computed. That is, the starting phase of the reference input sequence $PN_i^{(R)}(t)$ took values $0, 93T_c, \cdots, 930T_c$, corresponding to ($i = 1, 2, \cdots, 11$), where $i$ is the index of the convolver. Also, the phases of $PN_j(t)$, the input sequence, took values $0, T_c, 2T_c, \cdots, 92T_c$, corresponding to $j = 1, 2, 3, \cdots, 93$. Hence, $\rho_{ij}$ was calculated for all these combinations of $i$ and $j$ over the observation period $T$, and these $\rho_{ij}$ were then used to compute the corresponding values of $P_{ij}$.

There are four curves of $P_{ij}$ along with their corresponding values for $\rho_{ij}$ shown in Fig. 10. These curves correspond to ($i, j$) pairs of (2, 1), (3, 1), (4, 1), and (1, 3). With the notation introduced above, (2, 1) corresponds to the $P_{ij}$ of the second convolver when the starting phase of its input signal is zero and that of its reference is $93T_c$. Similarly, (1, 3) corresponds to the first convolver when its input has a starting phase of $2T_c$ and its reference a starting phase of zero. The maximum variations in $P_{ij}$, shown by these curves at any given SNR, come very close to the largest variation in $P_{ij}$ when all possible ($i, j$) combinations are considered.

Also plotted on this figure is the curve corresponding to $\rho_{ij} = 0$ (i.e., $\frac{1}{2}e^{-(1/2)\mathrm{SNR}}$). Upon comparing these curves, it can be seen that the probability of error generated by approximating the $\rho_{ij}$ by zero falls below all the other curves and, indeed, can be shown analytically to be a lower bound on performance. Similarly, the worst-case performance can be shown to correspond to that set of subcodes whose cross correlation is maximum in absolute value. For the spreading sequence that we chose, curve "$d$" on Fig. 10 yields worst-case results.

The advantage of the scheme, of course, is that in $2T$ seconds, $MN$ phase positions are examined. In a standard serial search technique, examining $MN$ phase positions requires $MNT$ seconds, and so an improvement of a factor of $MN/2$ is achieved. To be more specific, consider a serial search scheme that accumulates all the correlation samples over the entire uncertainty range and then makes a decision on the basis of the largest sample. If the integration time for this latter procedure is $T$ seconds per phase position in the search mode and $NT$ seconds per phase position in the lock mode, then, assuming the search-lock strategy of Fig. 2 is employed, the mean dwell time is given by

$$\tau_2 = \frac{K(1-P_L)^2(1+P_s) + NP_s^2(2-P_L)}{(1-P_L)^2(1-P_s(1-P_s))}\, T \tag{25}$$

where, as before, $K$ is the total phase uncertainty.

For either the parallel convolver scheme or the serial search scheme described above, the average time to correct acquisition can be found as follows. Assume that the time spent in lock at an incorrect phase position each time a false lock occurs is given by $T_f$, where $T_f$ is given by (23) for the parallel convolver scheme and (25) for the serial search technique. Also, let $\hat{T}$ be the time spent making a decision at any step in the search mode. Note that $\hat{T} = KT$ for the serial search and $\hat{T} = 2T$ for the parallel scheme, assuming $J = 1$. Then the mean time to acquire is given by

$$\begin{aligned} T_{acq} &= \hat{T}(1-P_e^{(s)}) + (T_f + 2\hat{T})(1-P_e^{(s)})P_e^{(s)} \\ &\quad + (2T_f + 3\hat{T})(1-P_e^{(s)})(P_e^{(s)})^2 \\ &\quad + \cdots + [mT_f + (m+1)\hat{T}](1-P_e^{(s)}) \\ &\quad \cdot [P_e^{(s)}]^m + \cdots \\ &= \frac{\hat{T} + T_f P_e^{(s)}}{1-P_e^{(s)}} \end{aligned} \tag{26}$$

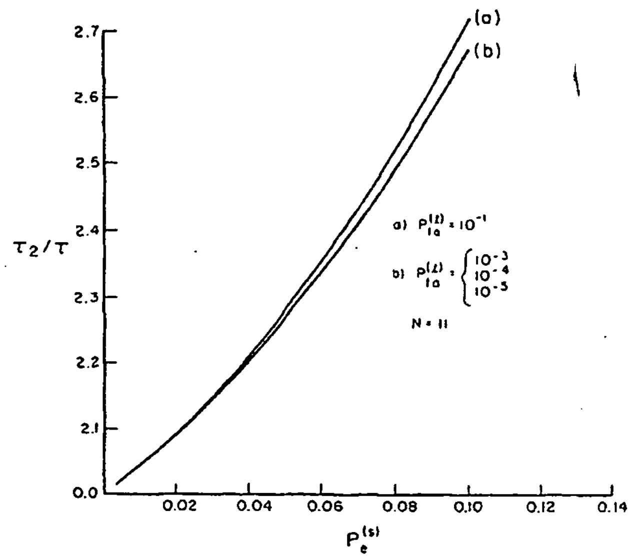


Fig. 11. Mean dwell time versus the false alarm probability in the search mode for various values of the false alarm probability in the lock mode.

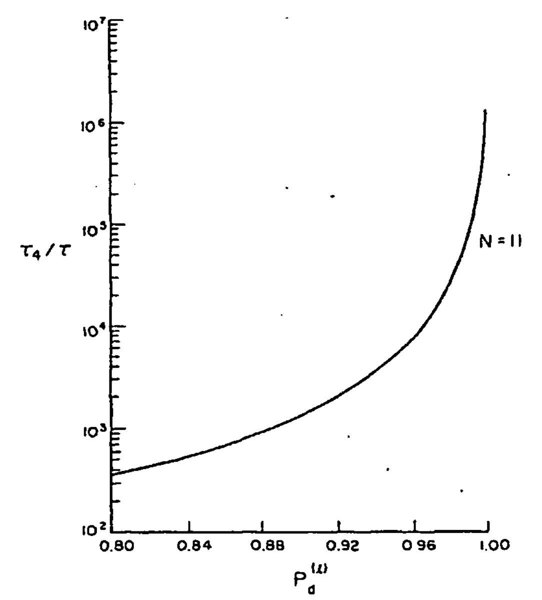


Fig. 12. Mean time to loss of lock versus the probability of correct detection in the lock mode.

Notice that if $T_f P_e^{(s)} \ll \hat{T}$, then

$$T_{acq}(\text{parallel convolver}) \cong \frac{2}{K} T_{acq}(\text{serial search}). \tag{27}$$

It is, of course, necessary to have an acceptable probability of error in the search mode, and if the $E/\eta_0$ required to yield a given level of performance is too large, one can always increase the number of states in the search mode. While this will not decrease $P_e^{(s)}$ on any given try, it *will* decrease the probability of entering lock at an incorrect phase position. Also, this further illustrates a point made in [4], namely, that a large integration time in the lock mode is desirable (recall that in the scheme described here, the integration time in the lock mode of any one convolver is the same as it is in the search mode, namely $T$ seconds, but that noncoherently summing $N$ output samples has the effect of yielding a larger integration time). Since one has to search *all* the phase positions in the uncertainty region in the search mode, but one only has to check a single phase position in the lock mode, once one has entered the lock mode, it is desirable to stay in that mode (i.e., not lose lock) for as long as possible. As long as the overall mechanism of entering the lock mode results in a correct decision with a reasonably high probability, the additional time spent in false lock caused by employing this strategy can be kept to a tolerable level.

Finally, it is at times more meaningful to consider such parameters as mean dwell time at an incorrect phase position and mean time to loss of lock at the correct phase position than it is to consider the probabilities of detection and false alarm. (Which set of parameters is more relevant to a given system is typically a function of how long the acquisition sequence is available, with the former parameters being more significant in continuously running sequences, as in certain navigation systems, and the latter parameters being more significant in more bursty systems, such as packet radios.) To illustrate the behavior of this particular acquisition scheme with respect to mean dwell time and mean time to loss of lock, consider Figs. 11 and 12. In Fig. 11, $\tau_2/T$ is plotted versus the false alarm probability in the search mode, with the false alarm probability in the lock mode as a parameter. In Fig. 12, $\tau_4/T$ is plotted versus the probability of correct detection in the lock mode.

## VI. Conclusion

A technique for the rapid acquisition of a DS spread-spectrum signal has been presented. The scheme relies on parallel processing different subcodes of a longer spreading sequence simultaneously in SAW convolvers, and results in a reduction of search time which is proportional to $M$ times $N$, where $M$ is the number of chips being processed in each convolver and $N$ is the number of convolvers. Results were presented for such key system parameters as probability of detection and mean time to loss of lock. While only analytical results were considered in the paper, it is believed that the system could be implemented with current technology in SAW devices.

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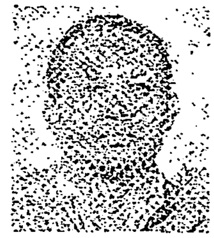

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